International Journal of Heat and Mass Transfer 98 (2016) 183–192

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Theoretically and numerically investigation about the novel evaluating standard for convective heat transfer enhancement based on the entransy theory

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ARTICLE INFO

Article history:
Received 15 August 2015
Received in revised form 29 December 2015
Accepted 24 February 2016
Available online 24 March 2016

Keywords:
Entransy theory
Energy dissipation
Efficiency factor
Field synergy principle

ABSTRACT

This paper theoretically and numerically investigated a novel evaluating standard δ_e for the enhancement of convective heat transfer based on the entransy theory. In the theoretical derivation, the differential equations about entransy are established using the method of micro-control volume which can give the obvious physical changing processes. And then, an efficiency factor δ_e is recommended according to physical mechanisms of entransy dissipation which is considered the impact of energy dissipation. The principle is verified by numerical simulation. The results indicated that δ_e is more appropriate for evaluating the ability of convective heat transfer enhancement comparing with the traditional method of FSP (field synergy principle) and EDEP (entransy dissipation extremum principle). The results also showed that the higher the factor of δ_e, the more efficient heat transfer process.

1. Introduction

Heat-exchange system is widely used in daily life, industry, energy utilization and so on. In recent years, it was used in aeronautics and astronautics also.

In order to design and optimize a heat-exchange system, some special and important parameters should be focused and calculated. Heat transfer coefficient and flow resistance were always coupled considered in design and optimization because that flow resistance determine the energy consumption of the system and the heat transfer coefficient reflect the profit of the system. At the same time, these two factors are coupled with each other to evaluate the energy utilization efficiency [1].

Lots of researchers paid attentions about how to reasonably evaluate the energy utilization efficiency of heat exchanger [2]. The entropy is one of the most popular factors to be used; it considered that the system is optimum if the system has the minimization of entropy generation [3–7]. Chen [8–13] pointed out that minimum entropy generation principle should be used for the target of reducing exergy loss. But the entropy generation represents the conversion of heat-to-work. Although the entropy and entransy are all corresponds to microstate number of the system [15], which indicates that both entropy and entransy could describe the irreversibility of thermal processes, the paradox would be derived by utilizing the minimum entropy for some flow and heat transfer optimization problems [39]. Thus, it is not suitable to evaluate the system without thermodynamic work because that the principle reducing the loss of exergy is not always equivalent to strengthen the process of the heat transfer system in the system without thermodynamic work.

The entransy theory is applied in heat transfer process optimization in recent years because that it can represent the heat transport potential capacity of an object [14]. Chen [8–13] also indicated that extremum principle of entransy dissipation adopted whereas for improving the heat transfer ability.

In aspect of theoretical analysis, Cheng [15–17] proved that the entransy has the property of the extensive quantity in the monatomic ideal gas system. At the same time, they investigated the changing of the entransy in isolated system and the results showed that the entransy is the uniform function for microscopic states and its value decreases during the process of the state transition. Following the work of Cheng [15–17], Meng [18–20] deduced the field coordination equation of the steady laminar flow heat transfer by utilizing the variation method.

In aspect of applied analysis, FSP (field synergy principle) and EDEP (entransy dissipation extremum principle) are two popular methods based on the entransy principle. Chen [21–24] optimized many heat transfer problems by using the EDEP. Xu [25] applied the minimum thermal resistance based on the entransy dissipation

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.02.081
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Nomenclature

t	time
h	static enthalpy
h_k	kinetic enthalpy
u	thermodynamic energy
e_u	entransy of thermodynamic energy per unit mass
e_h	entransy of enthalpy per unit mass
e_k	entransy of kinetic enthalpy per unit mass
$\dot{e}_q$	heat entransy flux
m, M	mass
c_v	specific heat capacity at constant volume
c_p	specific heat at constant pressure
T	static temperature
T_k	kinetic temperature
$T_h^{vir}, T_c^{vir}, T_{out}^{vir}, T_{in}^{vir}, T_{vis}^{vir}$	virtual temperature
$\Delta T_{hc}^{vir}, \Delta T_{oi}^{vir}$	virtual temperature difference
Q, Q_s, Q_t	heat flow
ΔE_c	entransy change over time in control volume
ΔE_{bou}	net entransy variation from boundary
E_{vis}	entransy caused by energy dissipation
E_{sour}	entransy caused by internal heat source
E_{diss}	entransy consumption rate
$\dot{m}$	mass flow rate
$\boldsymbol{q}$	heat flux
$\boldsymbol{v}$	velocity vector

Greek symbols

Φ	internal heat source
Ψ	energy dissipation
ρ	density
λ	heat conductivity coefficient
δ_e	efficient factor
τ	time

to optimize and design the heat exchanger structure. Feng [26] applied the EDEP to improve the global thermal insulation performance of thermal insulation constructer. He [27,28] also investigated the EDEP, and compared with the FSP. The result proved that the EDEP and FSP are inherently consistent for the convective heat transfer. Guo [29,30] proposed the novel concept of synergy angle to indicate the enhancing convective heat transfer of parabolic flow, and the results indicated that the convective heat transfer can be enhanced by reducing the intersection angle between the velocity and the temperature gradient. Tao [31–34] verified the rationality of synergy angle and extended the theory from parabolic flow to elliptic flow.

However, the method of EDEP and FSP has some constraint. Liu [35] mentioned that that the optimal solution of convective heat transfer may not be found without taking the energy dissipation into consideration. Minimum thermal resistance principle may not always be suitable to optimum the process of convection heat transfer because that minimum thermal resistance principle is generalized by heat conduction issues. At the same time, the consistent of FSP and EDEP is on the basis of statistic data instead of its physical mechanism.

According to the above introduction, there is no suitable factor or method to evaluate the enhancement of heat transfer when the energy dissipation is considered. This paper hopes to derive a factor to evaluate the enhancement of heat transfer based on the entransy theory. Based on the analogy between heat and mass transfer, the balance equation of entransy is established through theoretical derivation, the efficiency factor which takes the energy dissipation into consideration is recommended by analyzing the physical mechanism of integral equations. At last, a numerical example is provided to verify the applicability of the factor as the comparison function.

2. Entransy dissipation analysis in mass-control system

Based on the definition of entransy $E = \frac{1}{2}Q_{vh}T$, where Q_{vh} is the thermal energy stored in an object, three concepts about the entransy of thermodynamic energy per unit mass e_u, the entransy of enthalpy per unit mass e_h and the heat entransy flux $\dot{e}_q$ are given according to entransy theory [7] as follows

$$e_u = \int_0^T c_v T dT = \frac{1}{2} c_v T^2 \tag{1}$$

$$e_h = \int_0^T c_p T\, dT = \frac{1}{2} c_p T^2 \tag{2}$$

$$\dot{e}_q = \boldsymbol{q} T \tag{3}$$

The mass integral of e_u and e_h are the entransy stored in closed and open system respectively, and both of them are related to thermodynamic state [8]. The area integral of $\dot{e}_q$ is entransy migration from the boundary which happens during heat transfer process.

2.1. The entransy dissipation analysis for isolated system

An isolated system is shown in Fig. 1. There is no inner heat source and chemical reaction between any species of them. Three different kinds of gas with the different temperature (T_1, T_2, T_3), mass (m_1, m_2, m_3) and constant-volume specific heat (c_{v1}, c_{v2}, c_{v3}), are kept independently and then are mixed together suddenly. The temperature of system under thermal equilibrium is T. Assume that the temperature sequence from high to low is $T_3 > T_2 > T_1$, and all the constant-volume specific heat keep constant under different temperature.

For the isolate system, the energy conservation equation of the system is

$$c_{v1}m_1T_1 + c_{v2}m_2T_2 + c_{v3}m_3T_3 = (c_{v1}m_1 + c_{v2}m_2 + c_{v3}m_3)T \tag{4}$$

By the previous definition of the entransy of thermodynamic energy per unit mass e_u, the variation of entransy of each gas during the mixing process is

$$\Delta E_{u1} = m_1\left(e_{u1}^{before} - e_{u1}^{after}\right) = \frac{1}{2} m_1 c_{v1}(T_1^2 - T^2) \tag{5}$$

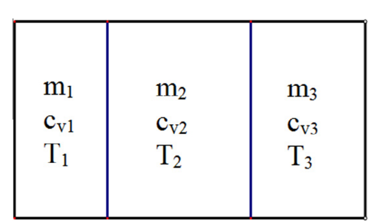

Fig. 1. The schematic of the isolated system.

$$\Delta E_{u2} = m_2\left(e_{u2}^{before} - e_{u2}^{after}\right) = \frac{1}{2}m_2c_{v2}(T_2^2 - T^2) \tag{6}$$

$$\Delta E_{u3} = m_3\left(e_{u3}^{before} - e_{u3}^{after}\right) = \frac{1}{2}m_3c_{v3}(T_3^2 - T^2) \tag{7}$$

The entransy changing of the overall system can be calculated by combing Eqs. (5)–(7) together as follows

$$\Delta E_u = \frac{1}{2}m_1c_{v1}(T_1^2 - T^2) + \frac{1}{2}m_2c_{v2}(T_2^2 - T^2) + \frac{1}{2}m_3c_{v3}(T_3^2 - T^2) \tag{8a}$$

The equation of Eq. (8a) indicates that the system entransy decrease is always positive because that

$$\frac{1}{2}\cdot\frac{c_{v1}m_1c_{v2}m_2(T_1-T_2)^2+c_{v1}m_1c_{v3}m_3(T_1-T_3)^2+c_{v2}m_2c_{v3}m_3(T_2-T_3)^2}{c_{v1}m_1+c_{v2}m_2+c_{v3}m_3}>0 \tag{8b}$$

Eq. (8b) can be extended into systems with more gas species from 3 to n, the entransy decrease is generalized as

$$\frac{1}{2}\cdot\frac{\sum_{i\neq j}^{n}c_{vi}c_{vj}m_im_j(T_i-T_j)^2}{\sum_{i=1}^{n}c_{vi}m_i} > 0 \tag{9}$$

It can be seen from Eq. (9) that the entransy variation is always decreasing when the gas with different temperature was mixed together under isolated condition. When the temperatures of gases (T_i) have the same value, Eq. (9) can be deduced into zero. It means that there is no entransy dissipation if the temperature of all the gases is equal. So the temperature difference may not only lead to heat flux but also is the essential power to generate the entransy dissipation.

2.2. Entransy analysis of mass-control system with boundary heat flow

A mass-control system which exchanges heat with the outside of the system is shown in Fig. 2. The system which is considered as the internal part is separated from the atmosphere using the virtual wall of the green circle which was shown in Fig. 2. The green circle is considered as the boundary of the system. The other part is considered as outside. The system is filled with ideal gas. Assume that the temperature of internal ideal gas inside the circle and the external environment are T_1 and T_2 respectively, $T_2 > T_1$. Total mass of the inside gas is M, and the specific heat c_v keeps constant. Total heat from outside to inside through the boundary is Q when the system reaches thermal equilibrium state with the outside.

Based on the analysis of isolated system, it can be inferred that the entransy dissipation is inevitable because of the uneven temperature. The heat Q is independent to the thermal process and always maintains constant under the same thermal boundary condition. Assume that all the thermal state changing is quasi-static process, and Q_s is the heat in each step of changing process. The equivalent model is established in Fig. 3 to analyze the entransy changing of the inside system. The heat transfer process is

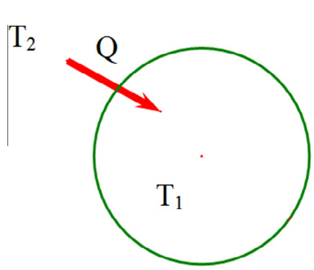

Fig. 2. The heat transfer model of the mass-control system.

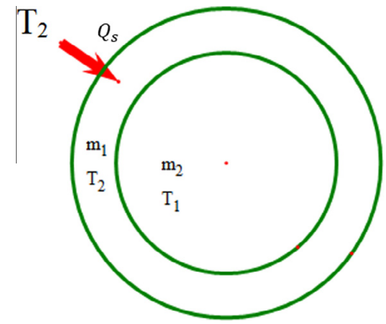

Fig. 3. Two step of heat transfer process model.

separated into two steps below, correspondingly the internal gas is divided into two parts denoted as m_1 and m_2. The first part of gas with m_1 absorbed the heat Q_s firstly. During that time, the temperature of the other part is constant. And then, the first part of gas and the second part of gas mix together. The process can be divided into two steps:

Step 1: Assume that the inside wall between m_1 and m_2 is adiabatic, the gas with m_1 is heated to T_2 firstly by transferring heat Q_s, in addition, Q_s can heat the temperature of the whole system up to T. The energy conservation gives

$$Q_s = c_vm_1(T_2 - T_1) = c_vM(T - T_1) \tag{10}$$

where M is the total mass of the system, and $M = m_1 + m_2$.

According the definition of e_u, the entransy increase of m_1 is

$$\Delta E_u(m_1) = m_1\left(e_{u1}^{after} - e_{u1}^{before}\right) = \frac{1}{2}c_vm_1(T_2^2 - T_1^2) \tag{11}$$

Step 2: Assume that the outside wall between m_1 and environment is adiabatic. Then, the gases of part one and part two spontaneous mix together. So the entransy dissipation is calculated as

$$\Delta E_u(m_1\_m_2) = \frac{c_vm_1m_2(T_2 - T_1)^2}{2M} \tag{12}$$

Combining the Eq. (11) with Eq. (12), the entransy dissipation of the system can be calculated as

$$\Delta E_u = \Delta E_u(m_1) + \Delta E_u(m_1\_m_2) \tag{13}$$

Eq. (13) can be written as

$$\Delta E_u = \frac{1}{2}c_vM(T - T_1)(T_2 + T_1) - \frac{1}{2}c_vm_2(T - T_1)(T_2 - T_1) \tag{13a}$$

According Eq. (10), Eq. (13a) can be rewritten as

$$\Delta E_u = \frac{1}{2}Q_s(T_2 + T_1) - \frac{1}{2}\frac{m_2}{M}Q_s(T_2 - T_1) > 0 \tag{13b}$$

where $Q_s(T_2 + T_1)/2$ is the total entransy transported into the system from the outside, $m_2Q_s(T_2 - T_1)/2M$ is positive and represents the entransy consumption rate during heat transfer process.

If the time of heat exchange is infinite until steady state, when the m_2 is close to zero and $Q_s$is to Q, Eq. (13b) can reduced into

$$\Delta E_u = \frac{1}{2}Q(T_2 + T_1) = \frac{1}{2}c_vM(T_2^2 - T_1^2) \tag{14}$$

Through the analysis mentioned above, it can be inferred that the entransy of heat is not merely promotes the entransy of gas, but also engenders the entransy dissipation during heat transfer process.

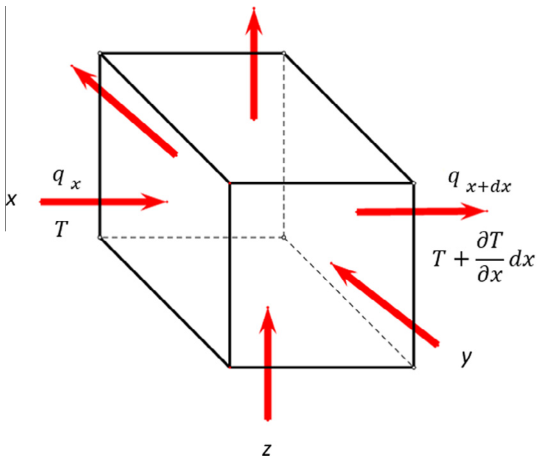

Fig. 4. Three-dimensional micro-body model.

3. Deduction of the entransy balance differential equations

Take the micro-control volume as shown in Fig. 4. Some physical quantities such as temperature and heat flux are graded distributed in micro-body. From the analysis in Sections 2.1 and 2.2, the entransy is changing because of unbalanced potentials, and the entransy dissipation always exists in heat transfer process. Ignore the chemical reaction hear and thermal radiation, the change of entransy for the system concludes four parts:

$$\Delta E_c = E_{bou} + \Delta E_{vis} + E_{sour} - E_{diss} \tag{15}$$

where ΔE_c is the total entransy change in control volume, ΔE_{bou} is the net entransy flow through the boundary, E_{vis} is defined as the analogic entransy source induced by viscous dissipated, E_{sour} is the entransy expression generated by the internal heat source, E_{diss} is the entransy dissipation during heat transfer process.

3.1. Derivation process of the heat conduction

The system in this section is considered as no mass transfer, so the entransy flow through the boundary is just induced by heat transfer, and $E_{vis} = 0$. ΔE_{bou}^{cond} can be expressed on the dimension of x according $\dot{e}_q$

$$\begin{aligned}\Delta E_{bou(x)}^{cond} &= \left(\dot{e}_{q(x)} - \dot{e}_{q(x+dx)}\right) dydzd\tau \\ &= q_x T dydzd\tau - q_{x+dx}\left(T + \frac{\partial T}{\partial x}dx\right) dydzd\tau \end{aligned} \tag{16a}$$

Base on the definition of heat flux, $q_x = -\lambda \frac{\partial T}{\partial x}$ Eq. (16a) can be written as

$$\begin{aligned}\Delta E_{bou(x)}^{cond} &= \left[-T\frac{\partial}{\partial x}\left(-\lambda\frac{\partial T}{\partial x}\right) - \lambda\left(-\frac{\partial T}{\partial x}\right)^2\right] dxdydzd\tau \\ &= -\frac{\partial}{\partial x}(q_x T) dxdydzd\tau \end{aligned} \tag{16b}$$

Similarly the dimension of y and z, $\Delta E_{bou}^{cond} = \Delta E_{bou(x)}^{cond} + \Delta E_{bou(y)}^{cond} + \Delta E_{bou(z)}^{cond}$. According that, ΔE_{bou}^{cond} can be represented by tensor as

$$\Delta E_{bou}^{cond} = -\nabla \cdot (\boldsymbol{q} T) dxdydzd\tau \tag{17}$$

Based on the analysis of Eq. (13b), the term of E_{diss} can be expressed on the x dimension

$$E_{diss(x)} = \left[q_x T - q_x\left(T + \frac{\partial T}{\partial x}dx\right)\right] dydzd\tau = \lambda\left(\frac{\partial T}{\partial x}\right)^2 dxdydzd\tau \tag{18}$$

Similarly with the definition of ΔE_{bou}, E_{diss} can be represented by tensor as

$$E_{diss} = -q \cdot \nabla T dxdydzd\tau \tag{19}$$

According the definition of entransy, changes of ΔE_c over time in micro-control volume can be described as

$$\Delta E_c = \rho \frac{\partial(\frac{1}{2}uT)}{\partial \tau} dxdydzd\tau = \rho c_v T \frac{\partial T}{\partial t} dxdydzd\tau \tag{20}$$

Assume that internal heat source is Φ. According the definition of entransy, the heat source causes the entransy is $E_{sour} = \Phi T d\tau$. Utilizing the method of tensor expresses, balance equation of the entransy can be calculated by integrating with Eqs. (17), (19) and (20), The relationship of Eq. (15) can be written as

$$\rho c_v T \frac{\partial T}{\partial \tau} = -\nabla \cdot (\boldsymbol{q} T) + \boldsymbol{q} \cdot \nabla T + \Phi T \tag{21}$$

Eq. (21) can be reduced to classical conservation equation if Φ is zero.

3.2. Derivation process of the convection heat transfer

Expect that the heat transfer makes contribution to the entransy changing. At the same time, the enthalpy transport also has an impact on the entransy in system. Besides, the heat generated by flow loss enlarges the entransy.

According the definition of e_h, the entransy of enthalpy which is contained in ΔE_{bou}^{flow} can be expressed as

$$\begin{aligned}\Delta E_{bou(x)}^{flow} &= \dot{m}_x\left(e_{h(x)} - e_{h(x+dx)}\right) d\tau \\ &= \frac{\rho v_x h_x T - \left(\rho + \frac{\partial \rho}{\partial x}dx\right)\left(v_x + \frac{\partial v_x}{\partial x}dx\right) h_{x+dx}\left(T + \frac{\partial T}{\partial x}dx\right)}{2} dydzd\tau \end{aligned} \tag{22a}$$

where $\dot{m}_x$ is the mass flow of x dimension. Eq. (22a) can be simplified as

$$\Delta E_{bou(x)}^{flow} = -\rho c_p T v_x \frac{\partial T}{\partial x} dxdydzd\tau \tag{22b}$$

where c_p is specific heat at constant pressure.

Similarly the dimension of y and z, flow component in $\Delta E_{bou}^{flow} = \Delta E_{bou(x)}^{flow} + \Delta E_{bou(y)}^{flow} + \Delta E_{bou(z)}^{flow}$ can be represented by tensor as

$$\Delta E_{bou}^{flow} = -\rho c_p T v \cdot \nabla T dxdydzd\tau \tag{23}$$

According to the relationship between enthalpy and thermodynamic energy, $h = u + p/\rho$, changes of ΔE_c over time in micro-control volume can be written as

$$\begin{aligned}\Delta E_c &= \rho \frac{\partial\left(\frac{1}{2}hT - \frac{1}{2}\frac{p}{\rho}T\right)}{\partial \tau} dxdydzd\tau \\ &= \left(\rho c_p T \frac{\partial T}{\partial \tau} + \rho T \frac{\partial(\frac{1}{2} v \cdot v)}{\partial \tau}\right) dxdydzd\tau \end{aligned} \tag{24}$$

Considering the compressible property of gas, the entransy variation caused by the change of kinetic energy should be taken into account. The kinetic energy is always expressed as kinetic enthalpy, entransy of kinetic energy can be derived based on the Eq. (2), formulated as

$$e_k = \int_0^T \left(\frac{1}{2} v \cdot v\right) dT = \int_0^T h_k dT = \int_0^T c_p T_k dT = c_p T T_k \tag{25}$$

where h_k is kinetic enthalpy per unit mass, T_k is equivalent temperature as called kinetic temperature. It is noticeable that T_k has no relevant to the integral temperature. Expanding will reduce the thermodynamic energy in control volume, thus, the entransy decrease by expanding under the previous temperature of x dimension is described as

$$\Delta E^{comp}_{bou(x)} = \dot{m}_x\left(e_{k(x)} - e_{k(x+dx)}\right)d\tau = \dot{m}_x T\left(c_p T_{k(x)} - c_p T_{k(x+dx)}\right)d\tau$$
$$= T\left[\rho\left(\frac{1}{2}v^2\right)v_x - \left(\rho + \frac{\partial\rho}{\partial x}dx\right)\frac{1}{2}\left(v^2 + \frac{\partial v^2}{\partial x}dx\right)\right.$$
$$\left.\times\left(v_x + \frac{\partial v_x}{\partial x}dx\right)\right]dydzd\tau \quad (26a)$$

Eq. (26a) can be simplified as

$$\Delta E^{comp}_{bou(x)} = -\rho v_x T\frac{\partial\left(\frac{1}{2}v^2\right)}{\partial x}dxdydzd\tau \quad (26b)$$

Similarly the dimension of y and z, the compressible issue $\Delta E^{comp}_{bou} = \Delta E^{comp}_{bou(x)} + \Delta E^{comp}_{bou(y)} + \Delta E^{comp}_{bou(z)}$ can be represented by tensor as

$$\Delta E^{comp}_{bou} = -\rho T v\cdot\nabla\left(\frac{1}{2}v\cdot v\right)dxdydzd\tau \quad (27)$$

Assume that the dissipated heat due to viscosity is Ψ, which causes the entransy is $E_{vis} = \Psi T d\tau$. Integrating Eqs. (23), (24) and (27) to modify Eq. (21) by choosing the same micro-control volume, the balance equation of entransy (Eq. (15)) for convection heat transfer is

$$\rho c_p T\frac{\partial T}{\partial\tau} + \rho T\frac{\partial\left(\frac{1}{2}v\cdot v\right)}{\partial\tau} = -\rho c_p T v\cdot\nabla T - \nabla\cdot(\boldsymbol{q}T) - \rho T v$$
$$\cdot\nabla\left(\frac{1}{2}v\cdot v\right) + \boldsymbol{q}\cdot\nabla T + \Phi T + \Psi T \quad (28a)$$

For incompressible flow with low Mach number, the item $\nabla\left(\frac{1}{2}v\cdot v\right)$ is negligible and $\rho = \text{constant}$. Then, Eq. (28a) can be written as

$$\rho c_p T\frac{\partial T}{\partial\tau} = -\nabla\cdot\left(\frac{1}{2}\rho c_p T^2 v\right) - \nabla\cdot(\boldsymbol{q}T) + \boldsymbol{q}\cdot\nabla T + \Phi T + \Psi T \quad (28b)$$

4. The efficiency factor for evaluating the enhancement of convective heat transfer

Enhancing the convective heat transfer and reducing the flow resistance are two important aspects to improve the performance of heat transfer equipment; however, they always contradict with each other, which imply that there exist some synergetic relations between them. In order to give a reasonable optimization goal for enhancing heat transfer, the physical mechanism of entransy balance equation should be considered again.

In the condition of incompressible gas with steady-state $(\partial T/\partial\tau = 0)$ and no internal heat sources $(\Phi = 0)$, the Eq. (28b) can be reduced into

$$-\boldsymbol{q}\cdot\nabla T = -\nabla\cdot(\boldsymbol{q}T) - \nabla\cdot\left(\frac{1}{2}\rho c_p T^2\boldsymbol{v}\right) + \Psi T \quad (29a)$$

Using Gauss transformation method, the control volume integral of Eq. (29a) can be expressed as

$$\iiint(-\boldsymbol{q}\cdot\nabla T)dv = -\oint\!\!\!\oint(\boldsymbol{q}T)d\boldsymbol{s} - \oint\!\!\!\oint\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}$$
$$+\iiint(\Psi T)dv \quad (29b)$$

where $\boldsymbol{v}$ is velocity vector, and $\boldsymbol{s}$ is area vector.

According the definition of Eq. (15), $\iiint(-\boldsymbol{q}\cdot\nabla T)dv$ equals to E_{diss}, $-\oint\!\!\!\oint(\boldsymbol{q}T)d\boldsymbol{s} - \oint\!\!\!\oint\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}$ equals to E_{bou} and $\iiint(\Psi T)dv$ equals to E_{vis}. Then Eq. (29b) can be reduced into

$$E_{diss} = \Delta E_{bou} + E_{vis} \quad (30)$$

Assume that Q_t is the total heat transfer rate between the internal fluid and the outside. The physical meaning of each item in Eq. (29b) will be analyzed.

The item $-\oint\!\!\!\oint(\boldsymbol{q}T)d\boldsymbol{s}$, represents the net heat entransy flow transported with the outside. $\boldsymbol{q}$ This item should range in appropriate values. For instance, as for boundary temperature T holds in constant (first boundary condition), heat transfer is enhanced ascribe to higher heat flux $\boldsymbol{q}$, therefor this item should maintain a higher value. Similarly, heat transfer is enhanced when the item keeps lower level in the condition of constant heat flux (second boundary condition). As for third boundary condition, the item changes in a reasonable region according to the limitation on the temperature or heat flux.

In order to simple the derivation, the new factor of virtual temperature is used in the following derivation

$$\Delta T^{vir}_{hc} = T^{vir}_h - T^{vir}_c = \frac{\iint_{in}(\boldsymbol{q}T)d\boldsymbol{s}}{Q_t} - \frac{\iint_{out}(\boldsymbol{q}T)d\boldsymbol{s}}{Q_t} \quad (31)$$

where T^{vir}_h is the virtual temperature of heat source, and T^{vir}_c is the virtual temperature of cold source. ΔT^{vir}_{hc} means the equivalent heat transfer temperature difference between heat source and cold source. The virtual temperature includes the total effect of temperature in three dimensions, it is traditional temperature difference in one-dimensional heat conduction. This item is correlation with the boundary conditions. Obviously, this value will remains constant under the first boundary conditions.

The item $\oint\!\!\!\oint\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}$, which represents the net enthalpy entransy flow around boundary. Based on the theory of convective heat field synergy [29,30], the non-dimension integral of $\bar{\boldsymbol{v}}\cdot\overline{\nabla T}$ is positive correlation with Nu, or, put another way, FSP is the different expression forms of Nu. It can be written as follows:

$$\bar{\boldsymbol{v}}\cdot\overline{\nabla T}d\bar{y} \propto Nu \quad (32)$$

The item $\oint\!\!\!\oint\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}$ can be transformed into $\iiint[\rho c_p T(\boldsymbol{v}\cdot\nabla T)]dxdydz$ by using the first Green's theorem. According to mean value theorem of integrals, there must exist a temperature $\tilde{T}$ to set the Eq. (33a) to hold

$$\iiint[\rho c_p T(\boldsymbol{v}\cdot\nabla T)]dxdydz = \tilde{T}\iiint[\rho c_p(\boldsymbol{v}\cdot\nabla T)]dxdydz \quad (33a)$$

Making Eq. (33a) non-dimension form as

$$c\iint\left(\mathrm{Re}\,Pr\int\bar{\boldsymbol{v}}\cdot\overline{\nabla T}d\bar{y}\right)dxdz \quad (33b)$$

where c is the constant number which does not affect the integral process. Combining Eqs. (32) and (33b), it can be inferred that the item $\oint\!\!\!\oint\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}$ is positive correlation with Nu. Thus, the higher this item, the better convection heat transfer enhancement process.

Introducing the following parameters

$$\Delta T^{vir}_{oi} = T^{vir}_{out} - T^{vir}_{in} = \frac{\iint_{out}\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}}{Q_t} - \frac{\iint_{in}\left(\frac{1}{2}\rho c_p\boldsymbol{v}T^2\right)d\boldsymbol{s}}{Q_t} \quad (34)$$

where T^{vir}_{in} is the virtual inlet temperature of fluid, T^{vir}_{out} is the virtual outlet temperature of fluid, ΔT^{vir}_{oi} represents the entransy variation of fluid transferring per unit heat. Considering heat transfer enhancement problem from the perspective of fluid, the issue can be equivalent to heat or cool the fluid. The higher absolute value this item represents the increasing for the degree of heat transfer potential capacity or decreasing per unit heat transferred. The higher value of ΔT_{oi}^{vir} also means the better heat transfer enhancement process under the same flow and heat boundary conditions.

The item $E_{vis} = \iiint(\Psi T)dv$, is the analogical entransy source induced by dissipated heat from fluid mechanical energy. It brings a positive effect by maintaining it a low proportion in entransy dissipation. Introducing the parameter T_{vis}^{vir} is

$$T_{vis}^{vir} = \frac{\iiint(\Psi T)dv}{Q_t} \tag{35}$$

The increase of heat transfer Q_t is always at the expense of boost of flow losses Ψ. This virtual temperature T_{vis}^{vir} represents the ability of weaken heat transfer enhancement which should keep relatively low level.

The item $E_{diss} = \iiint(-\boldsymbol{q} \cdot \nabla T)dv$ describes the loss of heat transfer ability. As mentioned above, ΔT_{hc}^{vir} is constant under the first boundary condition, heat transfer will be enhanced by higher value of $\Delta T_{oi}^{vir}$and lower T_{vis}^{vir}. Thus, with the same Q_t value, heat transfer performance will be strengthened for the small value of E_{diss}. In other words, the less entransy consumption per unit effective energy transferred, the more optimal heat transfer process will get.

A diagram of heat transfer model is shown in Fig. 5. Based on the above-mentioned analysis of Eqs. (31), (34) and (35), Eq. (29b) can be converted into

$$\frac{E_{diss}}{Q_t} = \frac{\iiint(-\boldsymbol{q} \cdot \nabla T)dv}{Q_t} = \Delta T_{hc}^{vir} - \Delta T_{oi}^{vir} + \Delta T_{vis}^{vir} > 0 \tag{36}$$

Under the first boundary condition, the higher value of ΔT_{oi}^{vir} and lower T_{vis}^{vir} is, the less consumptions E_{diss}/Q_t will get. It should stand out the impact of ΔT_{oi}^{vir} because that unlike the FSP, which describes the heat transfer intensity but not capacity, choosing the item ΔT_{oi}^{vir} to replace FSP (or Nu) is more appropriate to evaluate heat enhancement problems which kernel is potential changes but not superficial. Besides, the three items, ΔT_{hc}^{vir} ΔT_{oi}^{vir} and T_{vis}^{vir}, are interrelated and affected with each other. For instance, the item ΔT_{hc}^{vir} keeps constant under constant wall temperature boundary condition, raising ΔT_{oi}^{vir} will boost T_{vis}^{vir}, which lead to uncertainty changes of E_{diss}/Q_t for their reciprocal relationships. Thus, by comprehensive logical reasoning, the lower E_{diss}/Q_t, which is only the necessary condition, cannot indicate convective heat transfer enhancement. Therefore, the efficiency factor which is represented the non-dimension ratio of benefits and consumptions is recommend

$$\delta_e = \frac{\Delta T_{oi}^{vir}}{\frac{E_{diss}}{Q_t}} \tag{37}$$

Apparently, the higher this parameter is, the better convective heat transfer process will get.

5. Numerical validation

In the following presentation a numerical example was provided to validate δ_e, in addition, the comparisons of the different parameters such as FSP and EDEP [25,26] were also discussed details.

In the following discussion, the flow and heat transfer behaviors of air in the smooth tube and the step tube were investigated. In

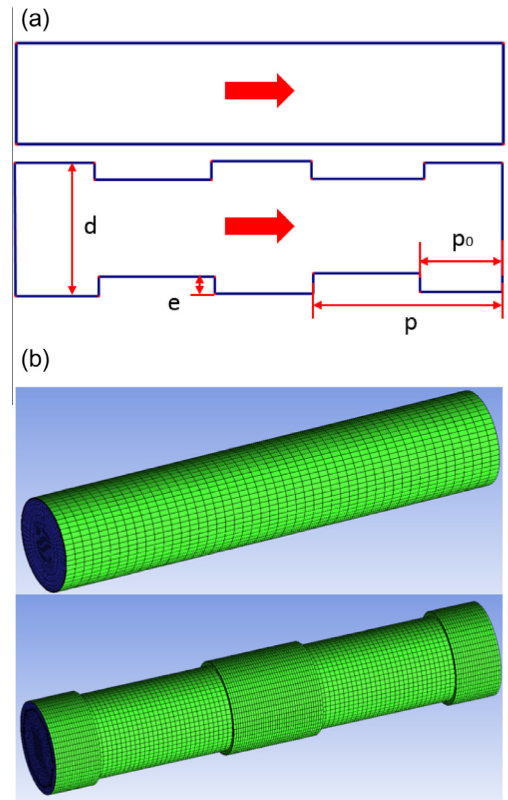

Fig. 6. Schematic describe of the numerical model with/without step (a) geometries (b) grid systems.

Table 1
Independent solution of mesh.

Total elements	Maximum yplus	Nusselt number	Deviation of *Nu* (%)
Smooth tube			
12,000	5.43	11.32	
26,600	2.97	11.94	5.48
45,000	0.93	12.38	3.68
77,650	0.37	12.41	0.242
Step tube			
42,400	2.72	17.25	
92,780	1.43	18.16	5.28
165,600	0.87	18.44	1.54
229,430	0.21	18.47	0.163

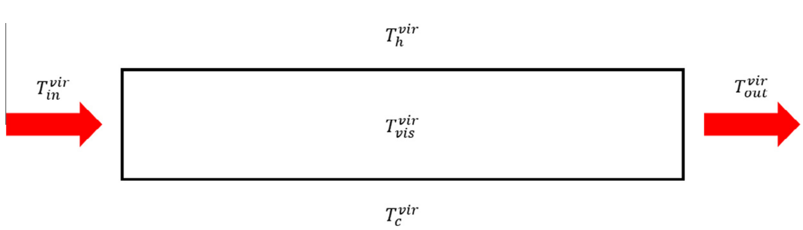

Fig. 5. Abstract heat transfer model.

the discussion, the flow is fully developed, the flow and heat transfer are in steady state and fluid thermo-physical properties are constants, besides, the wall temperature for both of the models is constant. The dimension of step tube is $e/d = 0.05$, $p/e = 50$, $p_0/p = 0.25$ [35,36]. The physical mean of e, d, and p were shown in Fig. 6. The grid system generated by ICEM is presented in

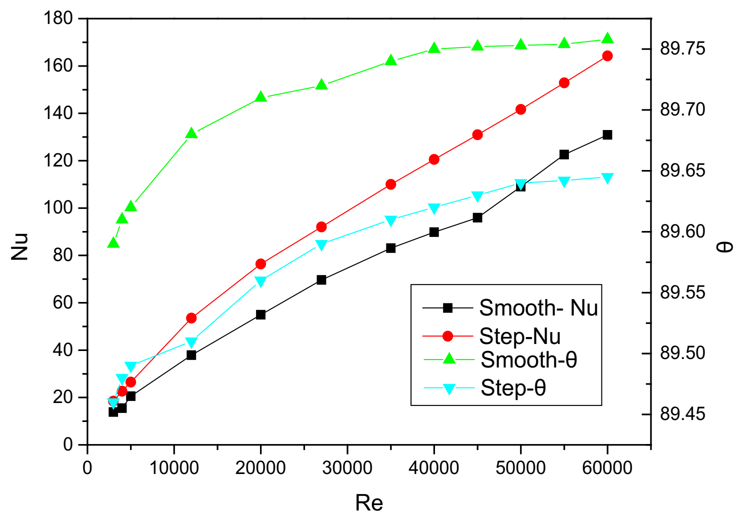

Fig. 7. Variation of *Nu* vs. and θ vs. with Re.

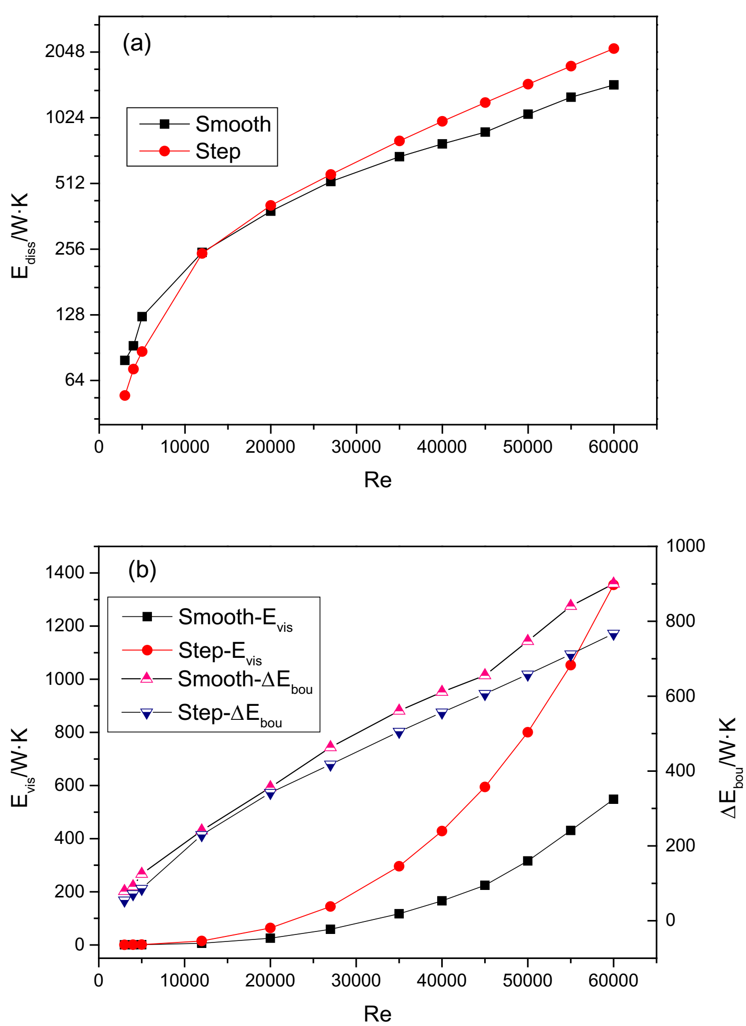

Fig. 8. Variation of (a) E_{diss} vs. and (b) E_{vis} vs. and ΔE_{bou} vs. with Re.

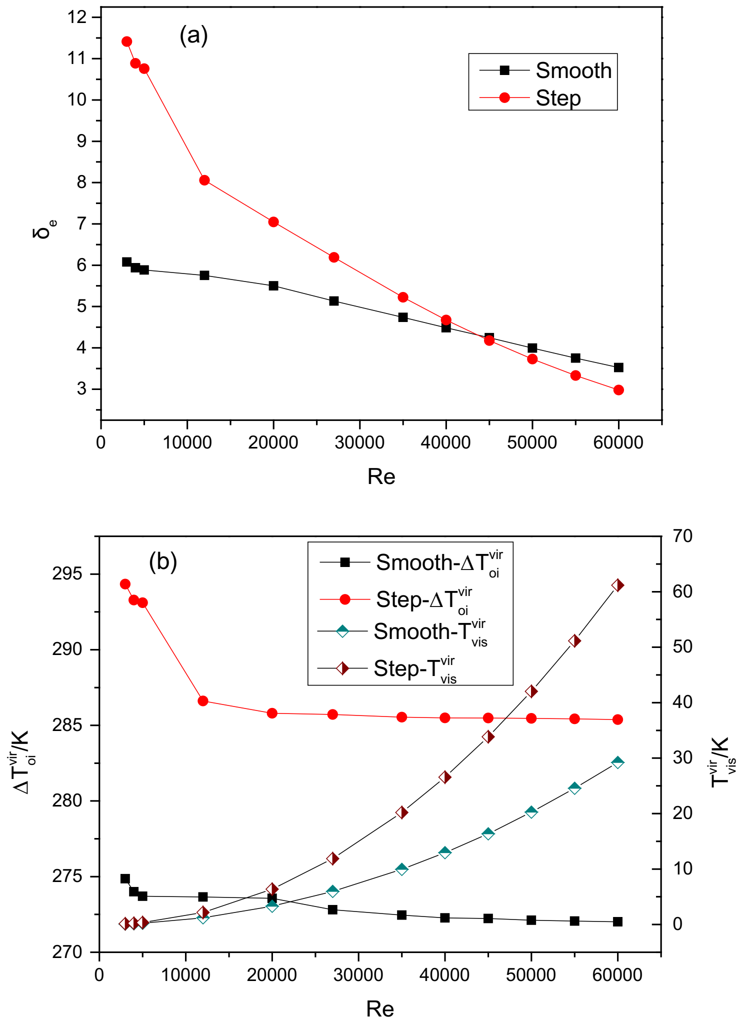

Fig. 9. Variation of (a) δ_e vs. and (b) ΔT_{oi}^{vir} vs. and T_{vis}^{vir} vs. with Re.

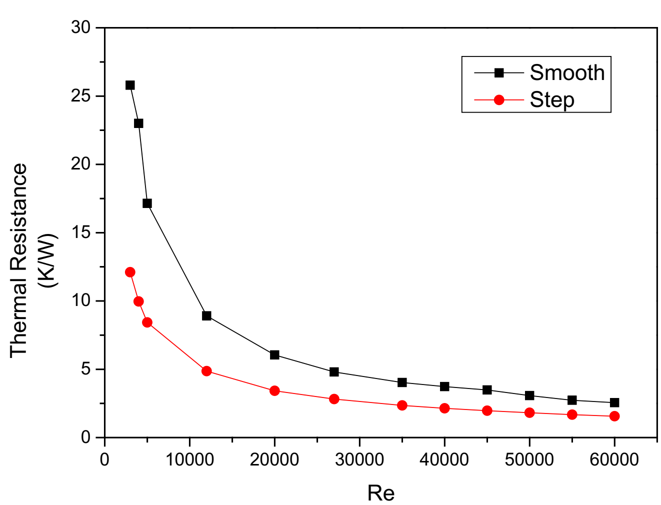

Fig. 10. Variation of thermal resistance.

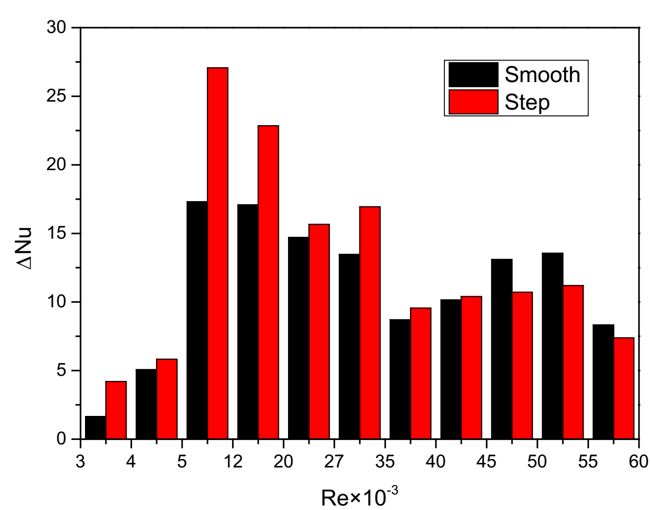

Fig. 11. The increment of *Nu* with Re.

Fig. 6(b), the smooth tube with total 45,000 hexahedral meshes and the step tube 165,600. The independent of mesh solutions is given as shown in Table 1. The criterion of grid independent solution is mainly considered two aspects, one is that the height of first layer should be applicative to low Reynolds number model (yplus < 1), the other is small difference of Nusselt number (deviation < 1%).

Utilizing k–ε model and SIMPLEC algorithms were adopted to deal with the linkage between velocity and pressure. Numerical solutions are conducted by using the software FLUENT. After the converged solutions are obtained, the domain averaged parameters above mentioned is determined by a UDF incorporated into FLUENT, and the characteristics of flow resistance and heat transfer for both of tubes are all verified by correlative empirical formula [36–38].

From the numerical results of temperature and velocity fields, the variations of Nusselt number $\left(Nu = \frac{Qd}{\lambda A(T_w - T_f)}\right)$ and FSP $(\theta = cos^{-1}\left(\frac{\sum \boldsymbol{v} \cdot \nabla T}{\sum |\boldsymbol{v}||\nabla T|}\right))$ with Reynolds number are shown in Fig. 7. It can be seen that, Nusselt number for the surface of step tube is higher than that of smooth tube; its synergy angle is lower than smooth tube.

The entransy of E_{diss}, E_{vis} and ΔE_{bou} in the process with Reynolds number are shown in Fig. 8(a) and (b), respectively. It can be observed from Fig. 8(a) that the entransy dissipation of step tube is lower than smooth tube when the Re is less than 12,000; the tendency is going into reverse when the Re is higher than 12,000. As is shown in Fig. 8(b), ΔE_{bou} for both of two tube increases linearly, and E_{vis} grows exponentially with Re. According to Eq. (30), it can be inferred that the impact of E_{vis} on E_{diss} can be neglected when the Re is lower than 12,000, after which is gradually becoming a dominant item with high Re.

The efficiency factor δ_e as well as the change of ΔT_{oi}^{vir} and T_{vis}^{vir} with Reynolds number is shown in Fig. 9(a) and (b). ΔT_{hc}^{vir} is constant for the given wall temperature case, ΔT_{oi}^{vir} of step tube is always higher than the smooth tube, also, T_{vis}^{vir} is getting higher with an exponentially growing. According to Eq. (37), it can be observed that δ_e of step tube is higher than smooth tube under the condition of low Reynolds number when the proportion of T_{vis}^{vir} is very small. However the Reynolds number is larger than 45,000, δ_e of step tube is lower than smooth, the major influence factor ΔT_{oi}^{vir} which is almost keeping constant is replaced by T_{vis}^{vir}. It can be indicated that the effect of the convective heat transfer enhancement of step tube becomes worse than smooth.

According to EDEP, the thermal resistance E_{diss}/Q_t^2, which principle is keeping this parameter to minimum, with Reynolds number is shown in Fig. 10. Obviously, the step tube is always better than smooth, which indicated that the thermal resistance cannot reflect the intersection point when Re is very high (45,000).

The increment of Nu in the same interval of Re are presented in Fig. 11. It can be observed that the growth rate of Nu for the step tube is faster than smooth before Re surpasses 45,000, the effect of heat transfer enhancement is approximate same around the region, after that, this enhancement is reversed.

It should be noted that the synergy angle line is always matching with Re, and there is no intersection point for these two lines, but δ_e has the intersection point. It inferred that the factor δ_e reflects efficiency of convection heat transfer when the energy dissipation is taken into account.

6. Conclusion

The new standards for the enhancement of heat transfer were introduced based on the balance difference equation of entransy is introduced by detailed theoretical inference which is considered the physical mechanism of heat transfer phenomena. According to analyzing each item in the integrated entransy differential balance equation of convective heat transfer, it is recommend that choosing the efficiency factor δ_e for evaluating convective heat transfer enhancement in which the impact of energy dissipation due to the shear stress is taken into consideration rather than the field synergy angle (FSP) and thermal resistance (EDEP). The higher this δ_e is, the better effect of the heat transfer enhancement will be. The numerical results indicate that the principle of the maximum δ_e is verified and its assessment ability is more effective than FSP, in addition, the entransy caused by viscous dissipation cannot be neglected in high Re region.

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