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Research Paper

A Monte Carlo simulation and effective thermal conductivity calculation for unidirectional fiber reinforced CMC

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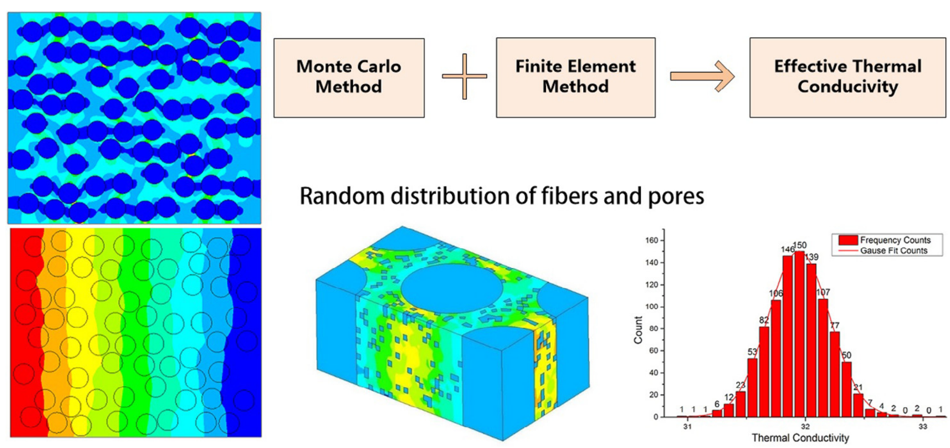
HIGHLIGHTS

- The random distribution of fibers and pores is simulated by Monte Carlo method.
- The effective thermal conductivity is calculated and the results are presented in statistical forms.
- The numerical procedure is programmed by APDL codes and executed automatically.

GRAPHICAL ABSTRACT



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ABSTRACT

Due to the high thermal-mechanical performance, ceramic matrix composite (CMC) is regarded as an alternate material for high temperature components of aircrafts. The thermal conductivity of unidirectional CMC is sensitive to its microstructural characteristics. Herein, the Monte Carlo method is introduced to simulate the real distribution of fibers and air pores. The complete procedure is programmed by ANSYS Parametric Design Language and executed automatically. The effective thermal conductivity of this composite at room temperature is calculated. The effect of fiber arrangement as well as matrix porosity with different fiber fractions is studied. The statistic results are gotten and both longitudinal and transverse thermal conductivity are computed. The random arrangement of fibers can result in a deviation of transverse thermal conductivity in different calculation times. The amount of air pores in matrix can also affect thermal conductivity of this material. This method is proved to be valid and accurate by comparison of numeric and experimental data; the relative error is less than 2%. The method can be used as an effective supplement of theoretical analysis and general FEM calculation for thermal conductivity.
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1. Introduction

Currently, a higher inlet temperature (more than 1900 K [1]) is required for gas turbine engine in order to increase thermal efficiency and reduce exhaust gas emission [2]. However, the traditional metal alloy (take k419 for instance, melt point: 1260~1340 °C, thermal conductivity: 10.65~26.80 W·m^{-1} K^{-1} [3]) is not suitable for that high temperature condition. Therefore it is urgent to find a new kind of alternative material.

Advanced ceramic matrix composite (CMC) is considered as a kind of viable candidate materials because of its higher temperature capability and lower density compared to metal alloy [4]. Among

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those, fiber reinforced ceramic matrix composites (FRCMC) is the most widely used CMC material [5]. When this material is used in stator and rotor blades, there is an increase of thermal efficiency and a decline of coolant consumption by raising turbine inlet temperature [6]. As a fundamental thermal characteristic, thermal conductivity determines the conduction capability of CMCs and is the basis of further research on thermal performance of whole blade.

The thermal conductivity of composites is sensitive to its microstructural details [7], such as fiber volume fraction, imperfections, porous, etc. The calculation research is attracting more attention with the development of technology and many different methods have been introduced. F. Gori evaluated the thermal conductivity of an ablative composite by theoretical method with a cubic cell [8]. C. Mahesh used finite element method to calculate the thermal conductivity of FRP composites with ANSYS software [9]. Marco Dondero applied Representative Volume Elements (RVE) and the Fast Multipole Boundary Element Method on the simulation of random micro-heterogeneous materials [10]. Giorgio Pia introduced fractal geometry method and heat resistance network on the computation of porous materials [11]. Ba Nghiep Nguyen brought in molecular dynamics (MD) method for analyzing thermal conductivity of SiC/SiC composites [12].

Generalized method of cell is widely used in the calculation of transverse thermal conductivity of unidirectional composites. It is an ideal model and is simplified by assuming that fibers are in regular arrangement. However, for most unidirectional FRCMC composites, the distribution of fibers over transverse cross section is usually random [13]. The description of structural details and the efficiency of numerical method is necessary for an accurate prediction [14]. Thus it is necessary to use a model that can reflect the structural characteristics for calculation.

The Monte Carlo (MC) method with a broadened model is used by many researchers to analyze the physical properties. T. Fiedler used the MC method for analysis of thermal diffusion in multiphase materials [15]. Chen X investigated the effect of random fiber arrangement on transverse permeability [16] as well as the distribution of interface stress [17]. A. Wongsto introduced this method to simulate the random distribution of fibers and research the mechanical properties on transverse cross section [13]. Zhao S Y used MC method to analyze the uncertainties quantification of thermal conductivity for ceramic fiber blanket [18]. This method is also used for simulating the distribution of air pores [19] and predicting thermal conductivity of porous media [20] as well as researching the effect of porosity [21].

Here, the objective of this paper is to introduce this method to calculate thermal conductivity of unidirectional FRCMCs, simulate the real distribution of fibers and pores under different fiber fractions, and research the rules and effects of microstructural characteristics on effective thermal conductivity with statistical information.

Herein, ANSYS Parametric Design Language (APDL) is used to program the calculation process. A set of complete APDL commands are coded for modeling, meshing, setting physical characteristics and boundary conditions, repeated solving, and data processing. The ANSYS Mechanical APDL software is used to execute the pre-generated macro document and then accomplish the whole procedure.

2. Theory

2.1. Heat conduction equation and boundary condition

For orthotropic CMC material, there are three main elements of thermal conductivity tensor, called main thermal conductivity. The heat conduction equation can be written as Eq. 1 [8]:

$$\begin{bmatrix} q_\xi \\ q_\eta \\ q_\zeta \end{bmatrix} = -\begin{bmatrix} \lambda_\xi & & \\ & \lambda_\eta & \\ & & \lambda_\zeta \end{bmatrix} \begin{bmatrix} \dfrac{\partial T}{\partial \xi} \\ \dfrac{\partial T}{\partial \eta} \\ \dfrac{\partial T}{\partial \zeta} \end{bmatrix} \tag{1}$$

Considering the area that heat flux passes through, the component of heat conduction equation along main directions can be described as:

$$\Phi_i = -\int \lambda_i \frac{dT}{dx_i} dA_i, \quad i = 1, 2, 3 \tag{2}$$

When thermal conductivity along x direction is to be calculated, certain boundary condition is set on the faces $x = 0$ and $x = l$, while the other faces are set to adiabatic. There are two kinds of boundary condition considered in this paper.

$$\text{Case 1:} \begin{cases} T|_{x=0} = T_1 \\ T|_{x=l} = T_2 \end{cases} \tag{3}$$

$$\text{Case 2:} \begin{cases} T|_{x=0} = T_1 \\ -\lambda A \left.\dfrac{dT}{dx}\right|_{x=l} = hA(T_f - T_w) \end{cases} \tag{4}$$

When the first boundary condition in case 1 is used, as is expressed in Eq. 3, the heat gradient is constant; then the effective thermal conductivity can be defined as:

$$\Phi = -\int \lambda \frac{dT}{dy} dA = -\bar{\lambda} A \frac{dT}{dy} \tag{5}$$

For FEM, the boundary face is meshed according to proper grid size. Consider the temperatures on two faces and the model length, the equation can be represented as the following form:

$$\lambda_{eff} = \bar{\lambda} = -\frac{\Phi}{A} \cdot \frac{l}{(T_2 - T_1)} = -\frac{\sum_{i=1}^{n} q_i a_i}{\sum_{i=1}^{n} a_i} \cdot \frac{l}{(T_2 - T_1)} \tag{6}$$

Here, a_i is the area of each mesh, and q_i is the heat flux density.

When boundary condition in case 2 is used, as is expressed in Eq. 4, temperature on the face $x = l$ is not uniform; thus, heat gradient is not constant. In this case, the effective thermal conductivity is defined as the weighted average of elements.

$$\lambda_{eff} = \bar{\lambda} = \frac{\sum_{i=1}^{n} a_i \lambda_i}{\sum_{i=1}^{n} a_i} \tag{7}$$

The thermal conductivity of each finite element can be written as:

$$\lambda_i = \frac{q_i}{\Delta T_i / \Delta x} = -\frac{q_i l}{T_{2,i} - T_1} \tag{8}$$

Then the effective thermal conductivity can be calculated by the following equation:

$$\lambda_{eff} = \bar{\lambda} = \frac{\sum_{i=1}^{n} a_i \lambda_i}{\sum_{i=1}^{n} a_i} = -\frac{l}{\sum_{i=1}^{n} a_i} \cdot \sum_{i=1}^{n} \frac{a_i q_i}{T_{2,i} - T_1} \tag{9}$$

2.2. Existing theoretical solving equations

Several theoretic models have been proposed to predict thermal conductivity of composites [22]. Among those, a simplified parallel and series model is widely used to calculate the effective thermal conductivity of unidirectional composites. The effective thermal conductivity along two directions can be calculated by Eq. 10 and Eq. 11 [23].

$$\lambda_L = (1-\phi)\lambda_m + \lambda_{fL}\phi \tag{10}$$

$$\lambda_T = \frac{\lambda_m \lambda_{fT}}{\phi\lambda_{fT} + (1-\phi)\lambda_m} \tag{11}$$

Here, ϕ is the volume fraction of fibers, L means the direction of longitude while T means transverse, f means fiber, and m means matrix.

For unidirectional fiber reinforced composites, the heat conduction in longitudinal direction fits parallel model well [24]. Thus the parallel model is used to calculate longitude results and compare with program solved data. However, in the transverse direction, the calculative results using series model can engender greater error; another simplified model and the total thermal resistor network [25] is used for theoretic analysis. The effective transverse thermal conductivity can be calculated by Eq. 12.

$$\lambda_T = \frac{\lambda_f\lambda_m\left(1-\sqrt{\phi}\right)^2 + \left(\lambda_m^2\left(1-\sqrt{\phi}\right) + \lambda_f\lambda_m\right)\sqrt{\phi}}{\lambda_f\left(1-\sqrt{\phi}\right) + \lambda_m\sqrt{\phi}} \tag{12}$$

2.3. The Monte Carlo method

The Monte Carlo method is also called stochastic simulation method. The approximate solution is given by calculating the statistic characteristics of parameters [26]. Assuming that the required amount x is the mathematical expectation of a random variable ξ, the approximate method is to repeat sampling for N times, then the arithmetic mean value can be calculated by Eq. 13.

$$x = E(\xi) \approx \frac{1}{N}\sum_{n=1}^{N}\xi_n \tag{13}$$

In this paper, the solution procedure is executed for given sampling times. After each time, a thermal conductivity result is generated, then the average value is calculated, and it represents the effective thermal conductivity.

3. Effect of fiber distribution

Herein, the MC method is used for simulating the real arrangement of fibers. The model contains a number of fibers, which is regarded as more realistic for analysis [17].

3.1. Modeling and solving process

The computational procedure consists three main processes, modeling the simulative distribution and meshing, setting boundary conditions and solving, and processing data. All these three processes together can be regarded as a sampling process and will be repeated for a given sampling times k. After each sampling time, a value of thermal conductivity will be generated. The amount of k should be big enough in order to get the accurate mathematical expectation.

The whole modeling procedure is represented as a flow chart in Fig. 1. There are some assumptions during the modeling and calculation process:

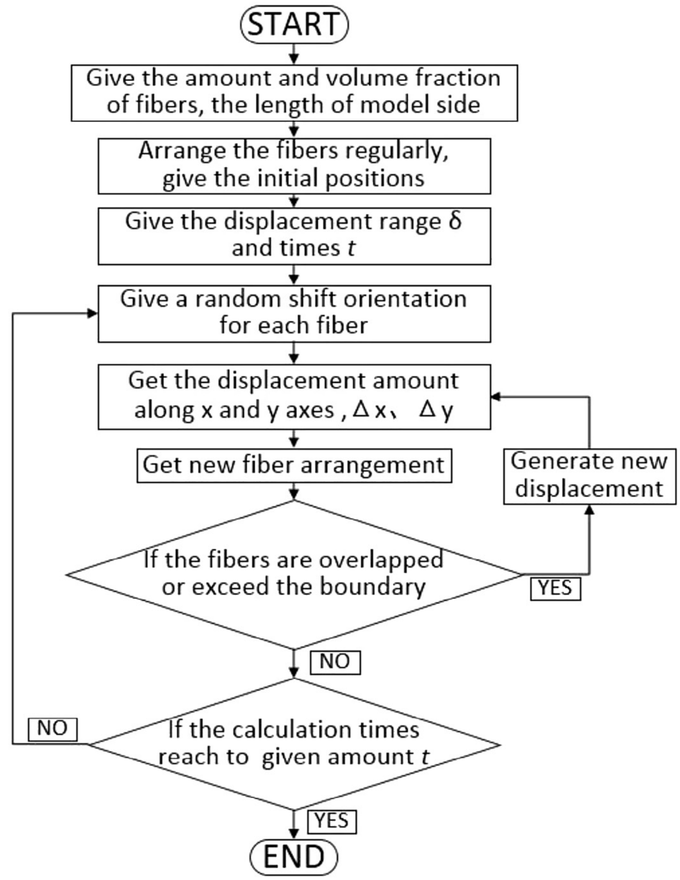


Fig. 1. Flowchart of MC modeling process.

1. All the fibers have the same value of diameter.
2. All the fibers are paralleled to the main axis.
3. Ignore defects in the material.

The fibers are first put regularly in the hexagonal array manner. The positions of fibers are determined by their coordinates along axes in cross section; here the transection face is the x–y plane. In order to get a new distribution, it needs to generate a series of coordinate values along x and y axes.

In this paper, fibers are perturbed for a sequence of sub-steps in order to get a fully dispersed distribution. For a single sub-step, each fiber moves with a minute displacement. The displacement is generated stochastically. The coordinate for a certain fiber at the t_{th} sub-step is (x_t, y_t), then the fiber moves to a new position (x_{t+1}, y_{t+1}).

The new coordinate can be presented as Eq. 14 and Eq. 15. Here, the function *random(m, n)* in ANSYS APDL is used to generate a random number between m and n.

$$x_{t+1} = x_t + random(0,\delta)\cdot\cos(random(0,2\pi)) \tag{14}$$

$$y_{t+1} = y_t + random(0,\delta)\cdot\sin(random(0,2\pi)) \tag{15}$$

After the generation of a new position, a judgment statement will be taken. A value of 0.1r is used as the gap limit between two neighbor fibers in many papers [13,17]. In this paper, the upper limit is set to 0.05r for gaps among fibers, as well as that between fibers and boundaries, in order to get the more real arrangement.

In this method, the distribution of fibers is determined by three factors: displacement range δ for each shift, the sampling times t and the volume fraction of fibers ϕ.

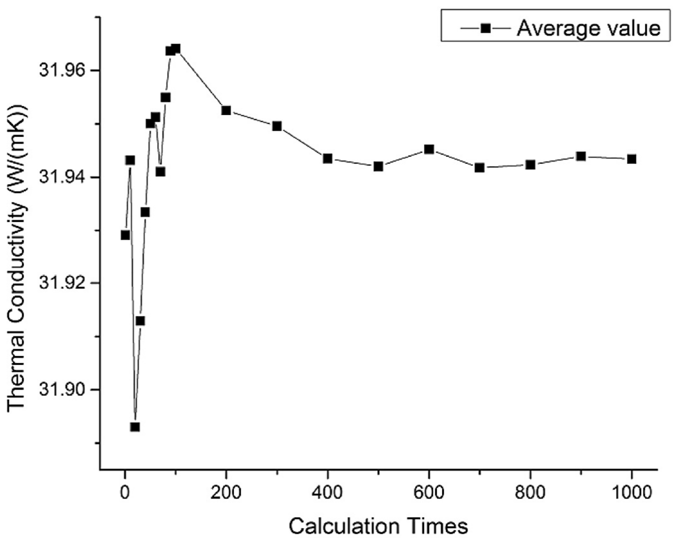


Fig. 2. Convergence of mean thermal conductivity for repeated calculations.

1. Given the constant value of δ and t, composites with a lower fiber fraction are relatively easier to generate dispersive distribution. For composites with high fiber fraction, the gaps among neighbor fibers are very narrow. Thus it is hard for fibers to move freely without overlaps and they can only shift to the limited area nearby. Thus the scattered effect is not obvious.
2. When t and fiber fraction ϕ are constant, the dispersion is becoming stronger as the increase of δ. Since the value of δ determines the range of fibers' displacements, a larger δ value means there is a higher possibility for fibers to move to further areas. Thus the distribution is more dispersive compared to the initial regular one.
3. Keep the values of ϕ and δ as constants, it is obvious that the distribution becomes more scattered with the growth of sub-step times. This is because the displacements can accumulate and the later generated positions are based on the previous ones. Thus if the number of sub-steps is large enough, the fibers can move to new positions far away from the original locations and then form a distinct arrangement compared to the ideal model [13].

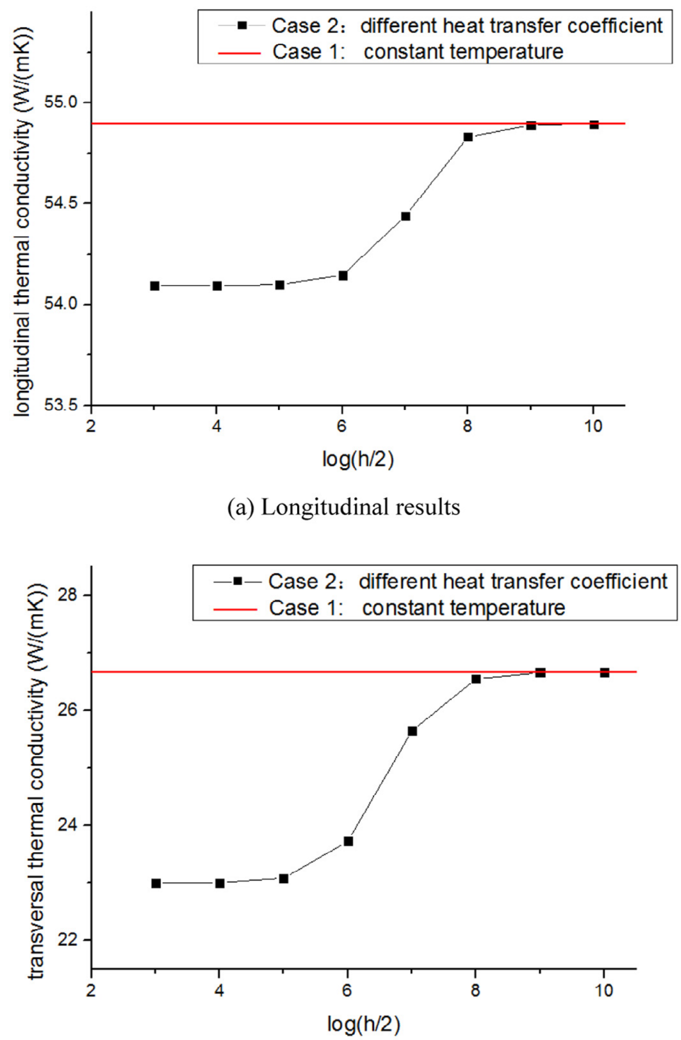
In this paper, $\delta = 0.1r$ and $t = 200$ are set in the process of calculation. Since the total time spent is increasing with the growth of t and k. This is a compromise between computational accuracy and is time consuming. The distribution of fibers is acceptable for numeration when t is greater than or equal to 200. The fiber fraction is set as a series from 20% to 70% in order to see the variations of properties with different components ratio.

The value of calculation times k is determined by the convergence of average value. As is shown in Fig. 2, the average value begins to converge when the number of calculation times is greater than 200. Thus the value $k = 200$ is used to get the precise numerical solutions.

In this paper, C_f/SiC is selected as one kind of CMCs for numerical analysis. The thermal conductivities in room temperature of carbon fiber on transverse and longitudinal directions are 4 W·m^{-1} K^{-1} and 40 W·m^{-1} K^{-1}, respectively; SiC matrix is 70 W·m^{-1} K^{-1} [27].

3.2. Discussion about boundary condition

Two kinds of boundary condition are set for model with fiber fraction of 50%. In case 1, the two boundary temperatures are set to 1000 K and 500 K, respectively. In case 2, one face is set with fixed temperature 1000 K, while on the other face the fluid temperature



(a) Longitudinal results

(b) Transverse results

Fig. 3. Numerical results of thermal conductivity under different boundary conditions.

is 500 K and a series of values for convection heat transfer coefficient h are set from 2000 W·m^{-2} K^{-1} to 2^{10} W·m^{-2} K^{-1}. The calculation results are represented in Fig. 3.

Although the fluid temperature T_f is constant, there is a huge difference among temperatures on face 2 for each state. With the growth of h, the temperature field on face 2 is getting more homogeneous, and the temperature on face 2 is approaching fluid temperature, from 999.954 K to 500.5 K. Thus the calculation result is approaching the data in case 1. When the value of h is big enough, there is no difference between the results of two cases.

The boundary condition in case 1 is used for the latter calculation of thermal conductivity in this paper.

3.3. Validation

As has been discussed in section 2, the effective thermal conductivity in longitudinal direction fits parallel model well. This model and Eq. 10 are widely used as a method to validate the calculation procedure [21]. A series of unit-cell models with fiber fraction from 20% to 70% are picked for numerical calculation, while Eq. 10 is used for theoretical analysis. The results indicate that the relative error between these two methods is less than 0.3%. Thus the ANSYS software is regarded as reliable for the calculation of thermal conductivity in this investigation.

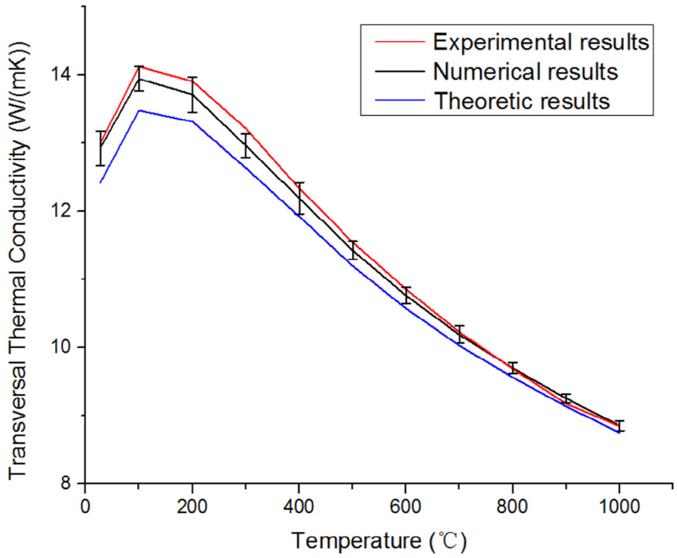


Fig. 4. Comparison of numerical, theoretic and experimental results.

Besides, in order to validate the accuracy of this method, the transverse thermal conductivity of composites in [28] is calculated by the MC method at various temperatures. The computed results are compared with measured data from that paper and theoretic analysis by Eq. 12. The comparison results are plotted in Fig. 4.

The comparison results indicate that the numerical results correspond with experimental results very well and the relative error is less than 2%, which can prove the validity and accuracy of this method. The accuracy of numerical results is higher than that of theoretic results.

3.4. Results and discussion

The random arrangement of fibers leads to the irregularity in both temperature field and heat flux density field, as is shown in Fig. 5. Due to the huge difference of thermal conductivity between matrix and fibers in transverse direction, heat is mainly transferred through the channel among neighbor fibers in matrix. Thus the heat flux density in matrix is much higher than that in fibers. In addition, the fibers can affect the heat conduction of nearby areas in matrix. Those portions that are close to fibers along heat transfer direction have the lower heat flux density than the other portions in matrix.

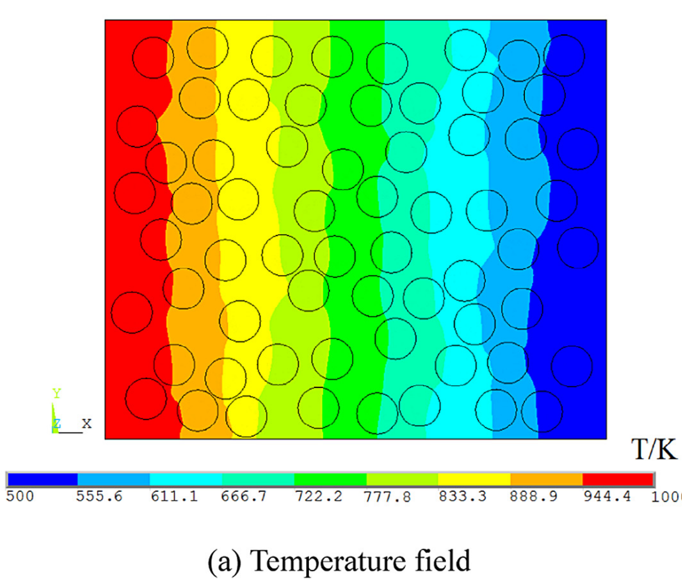
In order to get the statistical results of distribution as shown in Fig. 6, the model with the value $\phi = 40\%$ and $\delta = 0.1r$ is calculated for 1000 times. The computational results of effective thermal conductivity in vertical direction are scattered in the area between maximum and minimum values as shown in Fig. 6 (a). The offset (the difference between maximum and minimum values) is over 2 $W{\cdot}m^{-1}\,K^{-1}$, while most values are scattered nearby the mean value. And there is a decreasing possibility for values to distribute in the area far away from the average level. A fitting curve is drawn in the frequency count histogram. The frequency counts fit Gauss count very well. From the cumulative count figure, it can be seen that there are more than 90% of values located in the area of 32 ± 0.5 $W{\cdot}m^{-1}\,K^{-1}$.

The tendency of transverse thermal conductivity with the effect of fiber fraction is also studied as shown in Fig. 7.

Due to the random distribution, the values of thermal conductivity are in fact in a range between maximum and minimum. With Monte Carlo simulation and statistic result, the effect of fibers' arrangement can be noted.

Because fibers have much smaller thermal conductivity than matrix, the total effective thermal conductivity declines when fiber fraction increases. The deviation of maximum and minimum value is represented in Fig. 7 as well. The random arrangement of fibers can affect transverse thermal conductivity of this material. When fiber fraction is relatively low, there is an obvious difference between the maximum and minimum values; the deviation of maximum and minimum value can be as large as 2 $W{\cdot}m^{-1}\,K^{-1}$. The value of offset is less apparent with the increase of fiber fraction. This is because there is no enough space for fibers to move and generate a dispersed arrangement when fibers are crowded together under high fractions. In consequence, there are little differences of fiber arrangement among different calculation times.

Fiber fraction has a negative effect on the value of transverse thermal conductivity. When fiber fraction increases from 30% to 60%, the value of thermal conductivity decreases 44.4%, from 35.48 $W{\cdot}m^{-1}\,K^{-1}$ to 19.71 $W{\cdot}m^{-1}\,K^{-1}$. The numerical results are compared with theoretic results using Eq. 12. Numerical results fit theoretical data well and the error becomes much smaller with the growth of fiber fraction.



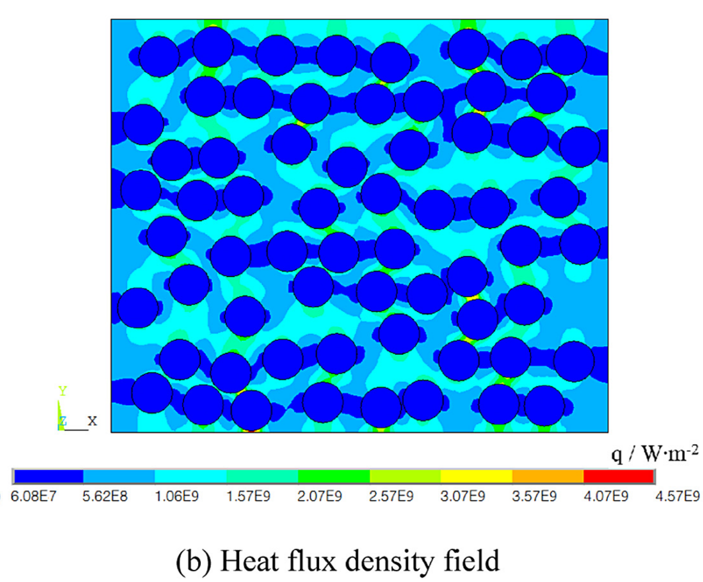
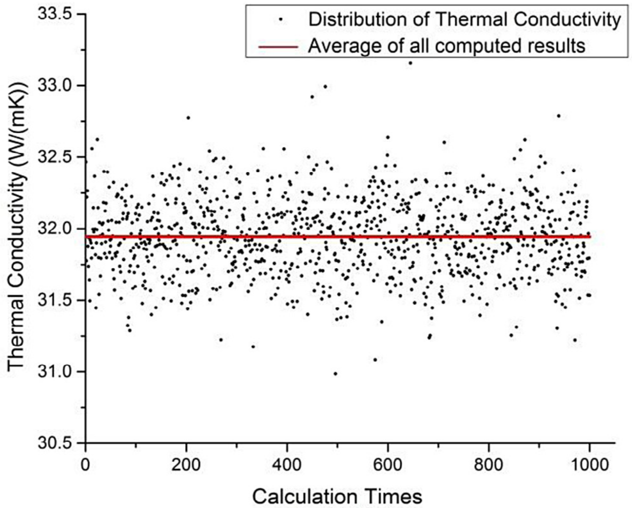
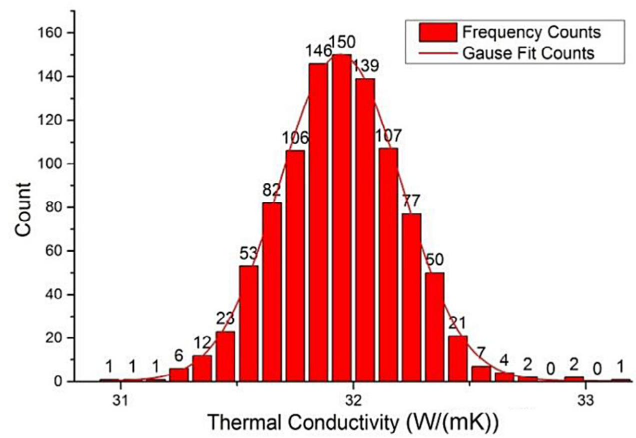


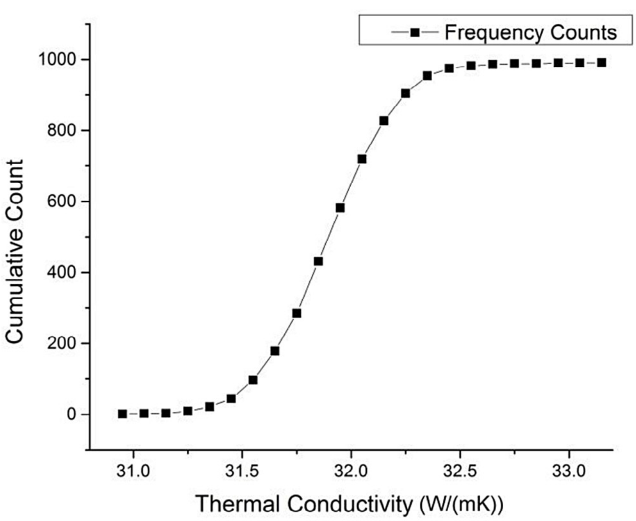
Fig. 5. Simulation results.



(a) Scatter distribution of computed results



(b) Frequency Count



(c) Cumulative Count

Fig. 6. Statistical results of thermal conductivity.

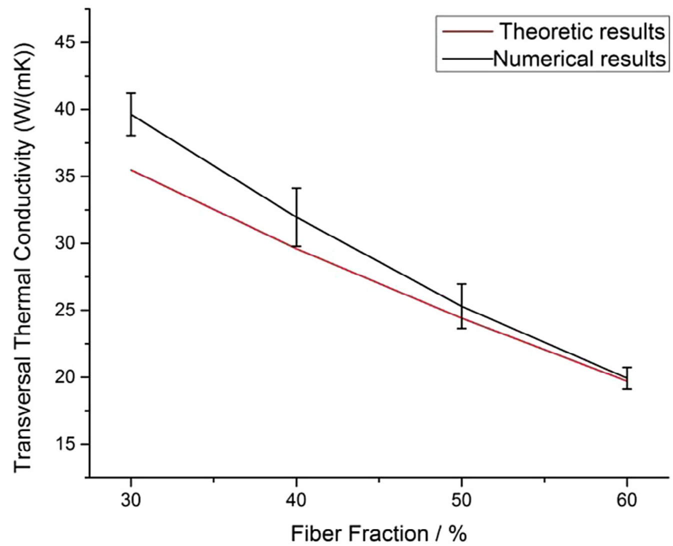


Fig. 7. Comparison of numerical results and theoretic results under different fiber fractions.

4. The effect of porosity in matrix

In the process of composites fabricating, the presence of air can lead to tiny air-filled cavities in the matrix. These pores are caused by the mixing and consolidation of discrete components [29]. In fact, most ceramic composites are porous [30]. As the thermal conductivity of air is far less than the thermal conductivity of matrix, the amount of pores will affect the effective thermal conductivity of composites [31].

The pores in the matrix can be regarded as randomly distributed. Thus Monte Carlo method is chosen to investigate the effect of pores on composites' thermal conductivity. The amount of pores in materials can be evaluated as porosity. For single-fiber CMC materials, the pores mainly exist in matrix, thus the porosity of matrix is used as a variable during the calculation process.

$$\phi_{pore} = \frac{\sum V_{pore}}{V_{matrix}} \tag{16}$$

4.1. Calculative procedure

A unit-cell model is chosen to calculate the thermal characteristics. Then the model is meshed in ANSYS APDL and the element type is set to SOLID 70.

Since the pores are distributed randomly in the matrix, a certain number of discrete elements are picked to represent these cavities. This process is achieved by the function *random(min, max)*. Here, *min* is the minimum number of elements in matrix, while *max* is the maximum number. After meshing, all the elements in the matrix have their own grid ID. So a series of random numbers without repetition are generated by this function. Each number corresponds to a pore element. The amount of selected elements n is determined by Eq. 17.

$$\sum_{i=1}^{n} V_i = \phi_{pore} \times V_{matrix} \tag{17}$$

Then the selected elements are assigned with thermal properties of air. The distribution of pores with matrix porosity 10% and fiber fraction 50% is presented in Fig. 8. Here, the yellow elements

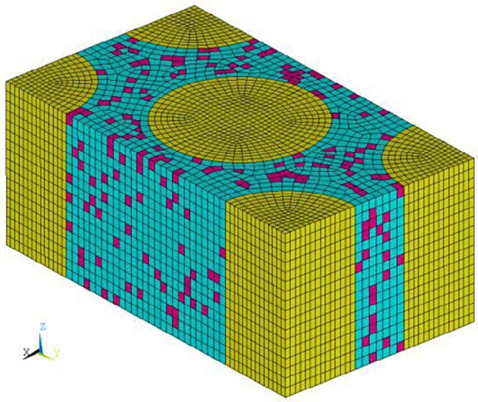


Fig. 8. Distribution of pores in matrix.

represent fibers, the blue elements represent matrix, and the purple elements are air pores distributed in the matrix randomly.

After that, the calculative process is executed by given times *k*. For each single process, a different pores arrangement is generated and a constant thermal boundary condition is given. After that, the thermal conductivities in both the longitude and transverse direction are calculated.

4.2. Results and discussion

For composites with the fiber fraction of 50% and matrix porosity of 15%, the temperature field and heat flux density field are shown in Fig. 9. The random distribution of pores leads to the irregularity in both temperature field and heat flux density field. In the matrix, there is little heat transmitted through the pores, due to the very small value of thermal conductivity of air [25].

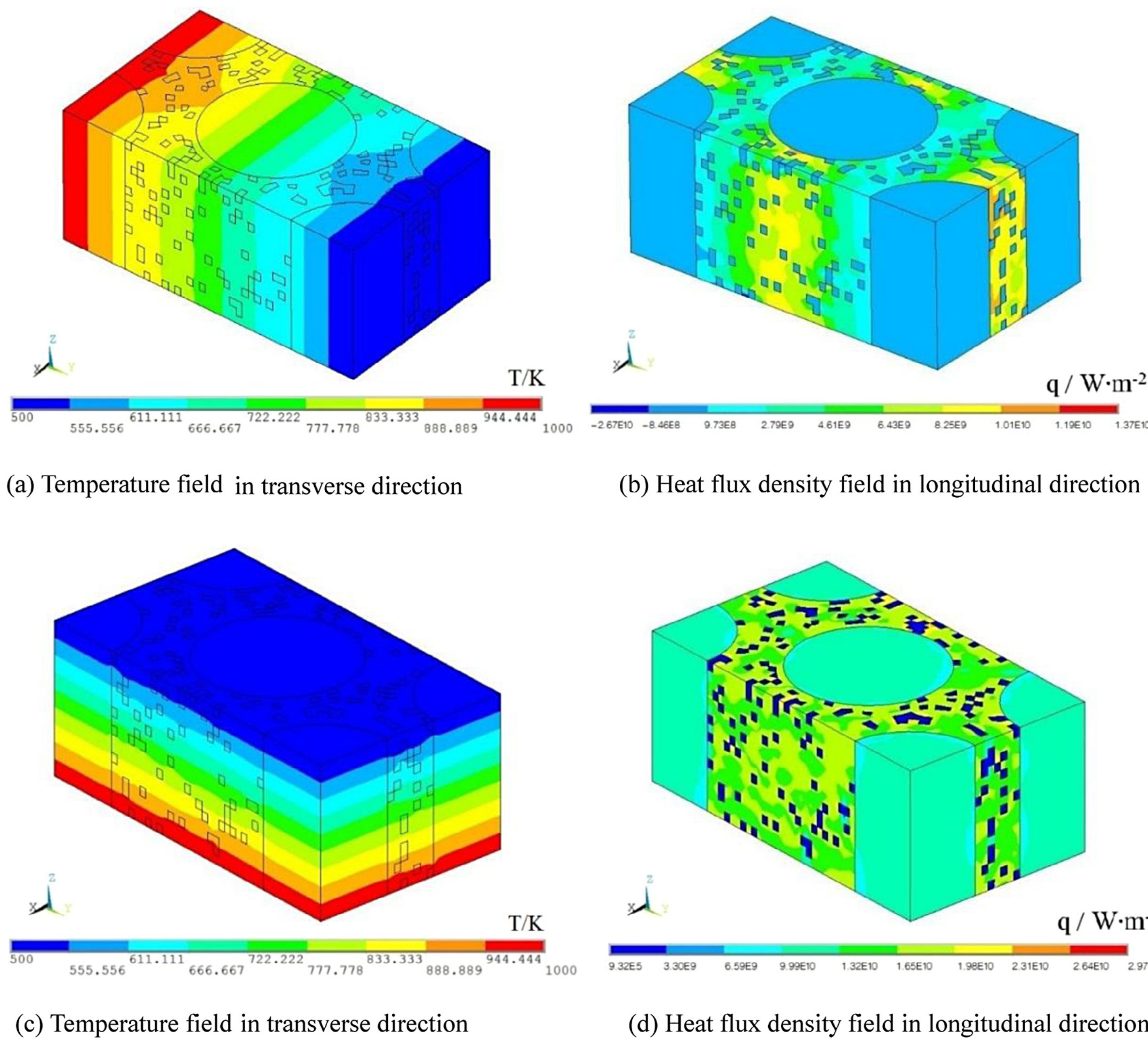
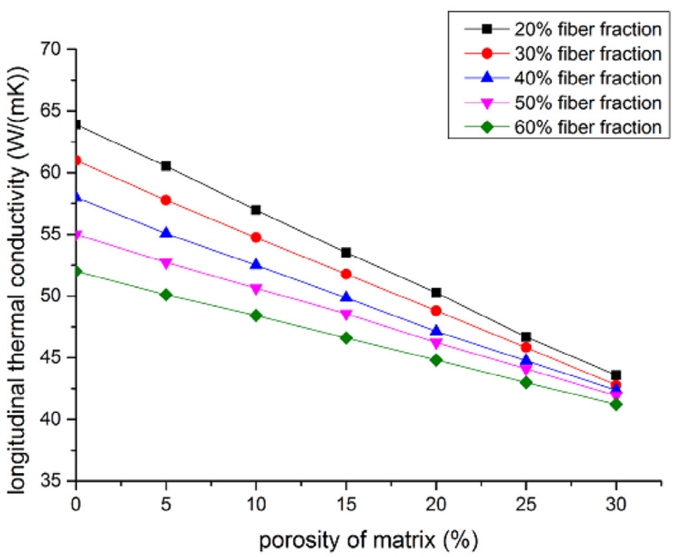


Fig. 9. Numerical results in transverse and longitudinal directions.

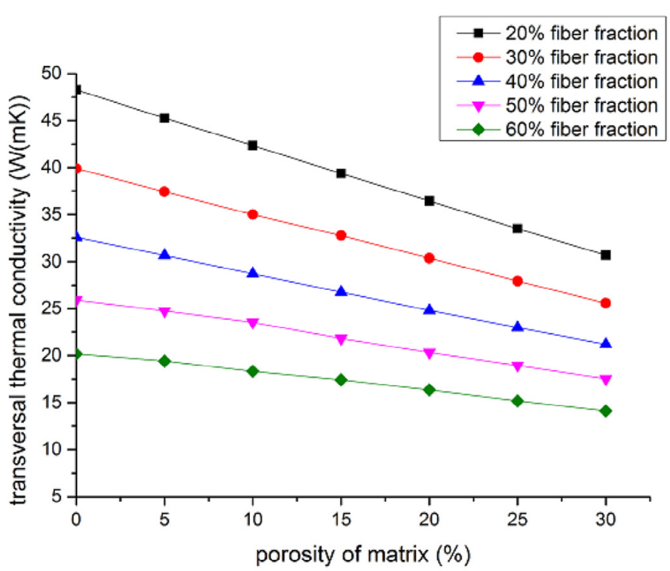
Table 1
Comparison between calculated data and theoretic data.

Porosity (%)	Transverse results			Longitudinal results		
	Numeric ($W \cdot m^{-1} K^{-1}$)	Theoretic ($W \cdot m^{-1} K^{-1}$)	Relative error (%)	Numeric ($W \cdot m^{-1} K^{-1}$)	Theoretic ($W \cdot m^{-1} K^{-1}$)	Relative error (%)
0	25.91	24.41	5.80	54.57	55.00	–0.78
5	24.76	23.38	5.56	52.72	53.25	–1.00
10	23.53	22.35	5.00	50.63	51.50	–1.69
15	21.81	21.32	2.26	48.55	49.75	–2.41
20	20.33	20.29	0.20	46.42	48.00	–3.30
25	18.94	19.25	–1.65	44.18	46.25	–4.47
30	17.54	18.22	–3.88	42.03	44.50	–5.56

The effect of pores' random distribution on the deviation of thermal conductivity in each calculation times is not obvious. The mean effective thermal conductivity of the model with different fiber fractions and matrix porosities is represented in Fig. 10. The value



(a) Longitudinal thermal conductivity



(b) Transverse thermal conductivity

Fig. 10. Change of thermal conductivity with porosity under different fiber fractions.

of each point is the average of repeated simulations. For both longitudinal and transverse directions, the effective thermal conductivity decreases with the growth of porosity in matrix. There is a 36.5% drop of transverse thermal conductivity at most and as most as 31.8% drop in longitudinal direction. The model with higher matrix fraction, namely smaller fiber fraction, has a bigger slope of decline. Therefore, pores have a negative effect on the effective thermal conductivity of CMCs.

For a model with a fiber fraction of 50%, the comparison between numerical results and theoretic results is listed in Table 1. The relative error between numeric and theoretic results is less than 6%.

In longitudinal simulations, the results for composites with small porosities fit the parallel model well and the relative error is very small. With the growth of matrix porosity, the composite structure of matrix and pores becomes more complex; the heat conduction through matrix and pores could not been explained by the simplified parallel model very well. Thus the error between two methods is increasing with the growth of matrix porosity. In addition, the longitudinal results have less relative errors than transverse data. This is because parallel model can explain longitudinal heat conduction for UD CMCs very well. However, in transverse direction, the theoretical formulas have poorer accuracy than in longitudinal direction.

5. Summary

1. In this paper, the Monte Carlo method is used to simulate the real distribution of fibers and pores in matrix. A complete calculative procedure is programmed by the ANSYS Parametric Design Language and is executed automatically. The validation of ANSYS software for thermal conductivity calculation has been proven.
2. The accuracy of this method is validated by the comparison of computational results and experimental data; the relative error is less than 2%, and the accuracy is higher than that of theoretic results.
3. The random arrangement of fibers can result in a deviation of effective thermal conductivity for composites in transverse direction. The influence is obvious when fiber fraction is less than 60%; the deviation of maximum and minimum value can be as large as 2 $W \cdot m^{-1} K^{-1}$.
4. Fiber fraction has a negative effect on the value of transverse thermal conductivity. When fiber fraction increases from 30% to 60%, the value of thermal conductivity decreases 44.4%, from 35.48 $W \cdot m^{-1} K^{-1}$ to 19.71 $W \cdot m^{-1} K^{-1}$.
5. The amount of air pores in matrix can also affect thermal conductivity of this material. The values in both longitudinal and transverse direction decrease with the growth of matrix porosity. There is a 36.5% drop of transverse thermal conductivity at most and as most as 31.8% drop in longitudinal direction. The relative error between numeric and theoretic results is less than 6%.

6. The implementation of this Monte Carlo method for analyzing the thermal conductivity of unidirectional fiber reinforced CMC has been introduced and demonstrated in this paper. The effect of random distributed fibers and the air pores in the matrix on effective thermal conductivity is investigated, and the effect of fiber fraction and porosity on thermal conductivity is researched. Depart from the unidirectional CMCs, this method also has the guidance significance for investigating thermal conductivity of other kinds of long unidirectional fiber reinforced composites with different matrix phases. The method with a broadened model can be regarded as a supplement of FEM with unit-cell model and theoretic analysis by empirical formula for researching heat conduction in transverse cross-section. For materials lacking experimental data, this method can offer a reliable prediction for thermal conductivity value.

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Nomenclature

CMC	Ceramic matrix composite
FRCMC	Fiber reinforced ceramic matrix composite
UD	Unidirectional
MC	Monte Carlo
FEM	Finite element method
RVE	Representative volume element
λ	Thermal conductivity [W·m^{-1} K^{-1}]
h	Convection heat transfer coefficient [W·m^{-2} K^{-1}]
λ_T	Transverse thermal conductivity [W·m^{-1} K^{-1}]
λ_L	Longitudinal thermal conductivity [W·m^{-1} K^{-1}]
ϕ	Fiber fraction [%]
d	Fiber diameter [μm]
r	Fiber radius [μm]
δ	Displacement range of each time [μm]
t	Sub-step times for modeling a single distribution of fibers
k	Repeat times for calculation [sampling times]
ϕ_{pore}	Matrix porosity [%]

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